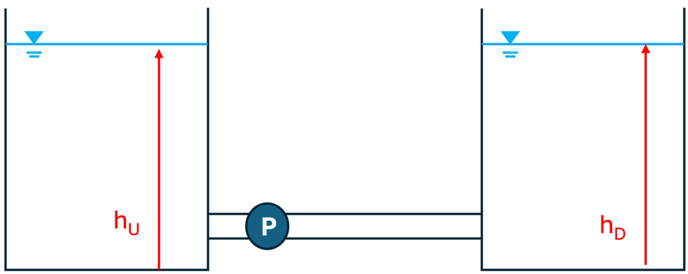
Pressurized Pipe Flow - Example



$D = 30$ cm
$L = 1$ Km
epsilon $= 0.6$ mm
$Q = 180$ l/s

P?

We have two reservoir for urban distribution connected by a pipe of diameter D and length L. The water level is the same in the two reservoirs. We need to convey a water discharge Q. The two reservoir are the same elevation z. Calculate the power needed for the pump.

We start from the Energy Balance equation:

$$H_D = H_U + h_P - h_T - h_L$$

where H = TOTAL HEAD $= \frac{P}{\gamma} + z + \frac{V^2}{2g}$

$U \Rightarrow$ upstream
$D \Rightarrow$ DOWNSTREAM

h_P = pump energy (ADDED)
h_T = turbine energy (SUBTRACTED)
h_L = head losses

$$\Rightarrow \underbrace{\cancel{z_D + \frac{P_D}{\gamma}}}_{h_D} + \cancelto{0}{\frac{V_D^2}{2g}} = \underbrace{\cancel{z_U + \frac{P_U}{\gamma}}}_{h_U} + \cancelto{0}{\frac{V_U^2}{2g}} + h_P - \cancel{h_T} - h_L$$

no turb

$h_D = h_U$

$$\Rightarrow h_P = h_L$$

pump energy is needed to win over the head loss

$$h_L = J \cdot L = \frac{f}{D}\frac{V^2}{2g}L$$

head Loss

We need to find the friction factor f

Colebrook & white

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{1}{3.71}\frac{\varepsilon}{D} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$f = \left[-2\log\left(\frac{1}{3.71}\frac{\varepsilon}{D} + \frac{2.51}{Re\sqrt{f}}\right)\right]^{-2}$$

We have ε and D, we need Re:

$$Re = \frac{VD}{\nu} \qquad \text{where } V = \frac{Q}{A} = \frac{Q}{\pi D^2/4}$$

$$\nu = 10^{-6}\ m^2/s$$

[$1\ m^3 = 1000\ l$]

$$= \frac{180 \cdot 10^{-3}\ m^3/s}{\pi \frac{0.3^2}{4}\ m^2}$$

$$= 2{,}55\ m/s$$

OK!

$$= \frac{2.55\ m/s \cdot 0.3\ m}{10^{-6}\ m^2/s}$$

$$= 7{,}64 \cdot 10^5 \Rightarrow \text{TURBULENT FLOW!}$$

Calculate the RELATIVE ROUGHNESS :

$$\varepsilon / D = \frac{6 \cdot 10^{-4}\ m}{0.3\ m} = 2 \cdot 10^{-3}$$

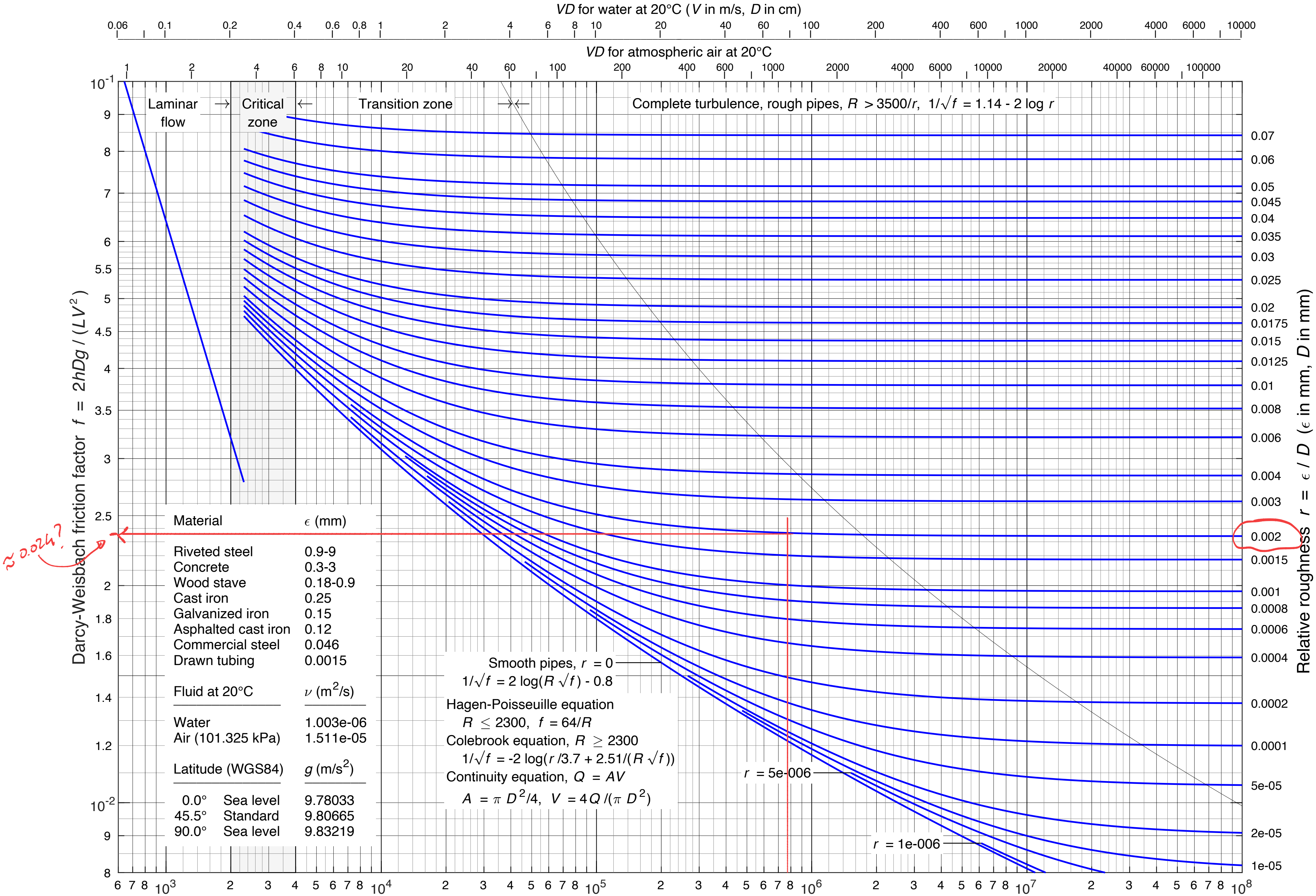
We have all the ingredient to calculate the resistance factor. We have two ways to do it:

1) We use C-&-W formula
2) we use the Moody diagram

Let's start graphically with the Moody diagram (see next page).

$\Rightarrow$ GRAFICALLY $\rightarrow$ $f \approx 0.024$

NOT QUITE, it's a bit less than that



Moody Diagram

Let's now do it numerically. Since f is on both sides of the equation, we have to get it by "trial and error", namely, iteratively.
To start the iteration, we always start with "wholly" turbulent flow assumption, so that we only use one part of the equation. In this case we are just a bit to the left of the black line separating the mix region from the wholly turbulent. So we make that assumption as a starting point and then we refine.

ITERATION I

$$f_I = \left[-2\log_{10}\left(\frac{1}{3.71}\frac{\varepsilon}{D}\right)\right] \quad \rightarrow \text{fully (wholly) turbulent or } Re \rightarrow \infty$$

$$= 0.0234$$

ITERATION II

We now adopt the full C&W expression using f_I

$$f_{II} = \left[-2\log_{10}\left(\frac{1}{3.71}\frac{\varepsilon}{D} + \frac{2.51}{Re\sqrt{f_I}}\right)\right]^{-2}$$

$$= 0.0236$$

Typically in 2 or 3 iterations you converge to a value.

Let's take f_{II} for valid $\Rightarrow$ $f = 0.0236$

We can now calculate the LOSSES!

$$h_L = \frac{f}{D}\frac{V^2}{2g} = 25,8 \text{ m}$$

$$\Downarrow$$

From the energy balance we found that $h_p = h_L$

$$h_p = h_L = 25,8 \text{ m}$$

I need to provide a head of 25,8 m with my pump. So that tells me the POWER of the pump I need:

From the energy balance equation we saw that the head supplied by pumps is:

$$h_p = \frac{1}{\dot{m} g} \frac{dW_p}{dt} \longrightarrow \boxed{h_p = \frac{P}{\rho Q g} = \frac{P}{\gamma Q}}$$

$\dot{m} = \rho V A = \rho Q$

dimensionally this is a POWER

$$\Rightarrow \quad P = \gamma h_p Q$$

$$= 9810 \frac{N}{m^3} \cdot 25,8\,m \cdot 0.18 \frac{m^3}{s}$$

$$\overbrace{N \cdot m}^{J} \cdot \frac{\cancel{m^3}}{s} \cdot \frac{1}{\cancel{m^3}}$$

$$\frac{\text{JOULE}}{s} = \text{WATT}\,!$$

$$= 45557,6 \text{ W} \longrightarrow \text{Watt}$$

$$= 45,5 \text{ kW}$$

If we want to know the energy required by the pump, we need to multiply its power by the time it operates

$$E = P \cdot t$$

Let's imagine it works for 10 hours:

$$E_p = 45,5 \text{ kW} \cdot 10 \text{ h}$$

$$= 455 \text{ kW} \cdot \text{h}$$

How much would that cost us?

⇒ COST OF ENERGY in LAUSANNE? ≈ CHF 0.41/kWh

↓

$$\text{COST POWER for } 10h = E_p \cdot c_e = 455\ \text{kWh} \cdot 0.41 \frac{\text{CHF}}{\text{kWh}} = 186{,}55\ \text{CHF}$$

What can we do to reduce the cost?

1) ↗↗ D
2) get a newer pipe (f ↓↓)
3) get a second pipe